Int. J. Production Economics 144 (2013) 301–315

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

A new multivariate gage R&R method for correlated characteristics

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ARTICLE INFO

Article history:
Received 17 July 2012
Accepted 22 February 2013
Available online 6 March 2013

Keywords:
Measurement system analysis
Repeatability and reproducibility
Principal component analysis
Roughness

ABSTRACT

This article explores how measurement systems having correlated characteristics are analyzed through studies of gage repeatability and reproducibility (GR&R). The main contribution of this research is the proposal of a method for multivariate analysis of a measurement system, a method that considers the weighted principal components (WPC). To prove its efficiency, what was first evaluated were the measurements of the roughness parameters obtained from AISI 12L14 steel turning machined with carbide tools. This GR&R study considers 12 parts, 3'operators, 4'replicates, and 5'responses (R_a, R_y, R_z, R_q and R_t). The data set has a correlation structure that determines 86.2% of explanation for the first principal component. As another step in proving the method's efficiency, the study generates simulated data with different correlation structures for measurement systems classified as acceptable, marginal, and unacceptable. The proposed method is compared with classical univariate and multivariate methods. It was observed that, compared to the other methods, the WPC was more robust in estimating the assessment indexes of a multivariate measurement system.

1. Introduction

Quality improvement projects are often characterized by their objective to reduce variability and achieve zero-defect production. If a product fails to conform to these standards, analysts generally blame the process and then act to improve process capability. In some instances, however, the process capability may be fine. Yet the measurement error, when compared to the variability of the process, remains unacceptable (Al-Refaie and Bata, 2010). Hence, before a team of analysts tries to improve a process, they should investigate both the variability of the measurement process as well as the variability of the manufacturing process.

Many manufacturers today use process capability to judge a supplier's ability to deliver quality products (Wu et al., 2009). However, there is no capability unless the process is under control. Control charts are commonly used to analyze whether a process is stable. Only once a process is under control statistically (that is, producing consistently), determination of its capability is required. This means analysts determine whether it is meeting specification limits and producing "good" parts. If the data include correlated variables, analysts could be misled if they use univariate techniques. After all, the variables jointly affect the process. For instance, if analysts employ separate univariate control charts to track a multivariate situation, they will face a Type I error. The probability of their plotting a point correctly as being under control is not equal to the points' expected values. The distortion of these values increases with the number of measurement variables. The literature contains many studies that analyze, using control charts and process capability indexes, manufacturing processes having multiple characteristics. Some of these include: Chen et al. (2005), Villalobos et al. (2005), Yang and Rahim (2005), Chen et al. (2006), Pan (2007), Pan and Jarrett (2007), Machado and Costa (2008), Chen and Chen (2008), Psarakis (2011), Boone and Chakraborti (2012), and Wang (2012). What is lacking in the literature, however, research on measurement system analysis with multiple characteristics (Wang and Chien, 2010).

To draw inferences regarding products and process quality, manufacturers use quantitative methods. Such methods (e.g., process capability indexes and control charts) incorporate data into the decision-making process. Prior to obtaining data, a manufacturer should help to ensure its validity by evaluating the measuring device (Majeske, 2008). According to Wu et al. (2009), the inevitable variations in process measurements arise from two sources: the manufacturing process and the gage. In manufacturing, a measurement system is not used to produce an exact dimension of a part. Such a system provides measurements

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http://dx.doi.org/10.1016/j.ijpe.2013.02.018

that, due to errors (random and systematic), vary from the true value (AIAG, 2010). In any activity involving measurements, whatever observed variability not due to the product/process itself,σ_P^2, is due to measurement error or variability in the measurement system, σ_{MS}^2 (Costa et al., 2005; Li and Al-Refaie, 2008; Senol, 2004; Woodall and Borror, 2008).

To identify the components of variations in the precision and accuracy assessments of measuring instruments, researchers often rely on measurement system analysis (MSA). The purposes of MSA are to: (1) determine the extent of the observed variability caused by a test instrument, (2) identify the sources of variability in a testing system, and (3) assess the capability of a test instrument (Burdick et al., 2003). According to He et al. (2011), MSA is an important element of Six Sigma as well as of the ISO/TS 16949 standards. GR&R is the most common study in MSA to evaluate statistical variations in the measurement process. Repeatability is the variation in measurements obtained by one measuring instrument when used several times by one appraiser while measuring an identical characteristic on the same part. Reproducibility is the variation in the average of measurements made by different appraisers using the same gage when measuring a characteristic on one part (Awad et al., 2009; Burdick et al., 2003; Erdmann et al., 2010; Polini and Turchetta, 2004; Van Den Heuvel and Trip, 2002; Wu et al., 2009). GR&R aims to determine that a measurement system's variability is less than that of the monitoring process (Al-Refaie and Bata, 2010; Shiau, 2000; Wang and Chien, 2010).

1.1. Univariate approach for GR&R studies

As emphasized above, a team of analysts, before analyzing the process capability of a quality improvement project, should evaluate the capability of the measurement system. Two methods commonly used in the analysis of a GR&R study are: (1) an analysis of variance (ANOVA) approach followed by estimation of the appropriate variance components; and (2) an X-bar and Range chart that estimates the standard deviations of the components of gage variability (Wang and Chien, 2010). Analysts prefer the ANOVA method because it measures the operator-to-part interaction gage error; this variation is not included in the X-bar and Range method (AIAG, 2010). Burdick et al. (2003) provided a good review of methods for conducting and analyzing measurement system capability studies, which are based on the analysis of variance approach. Dejaegher et al. (2006) used Six Sigma to measure, analyze, and improve the capability of a procedure required in the testing of the quality of an active pharmaceutical ingredient. This was done using multiple GR&R studies to analyze the capability of the measurement procedure. A design of experiments was next designed to improve this procedure.

Kaija et al. (2010) used the Six Sigma DMAIC (define, measure, analyze, improve, control) approach to evaluate a process of printing a dielectric layer with an inkjet printer. Initially, a GR&R study was conducted to evaluate the proportion of variation caused by the measurement system and process variation. Experiments were then planned and analyzed to identify the parameters having the most significant effects on the output variables of the dielectric layer's insulating layer and surface roughness. Li and Al-Refaie (2008) used the Six Sigma DMAIC procedure to improve quality through enhancing the measuring system capability of the wood industry. The measurement system assessed through GR&R had been considered unacceptable. To improve it, analysts implemented corrective actions, including operator training, proper selection of measuring instruments, and improved measuring procedures. In a second GR&R study, the authors concluded that the corrective actions had reduced the *%R&R* index (percentage of repeatability and reproducibility) by 39.38% and had improved the *ndc* index (number of distinct categories) by 168.84%.

Other studies have employed GR&R to evaluate measurement systems. In proposing a procedure to evaluate measurement systems and process capabilities, Al-Refaie and Bata (2010) used GR&R along with four quality measures. The quality measures were: precision-to-tolerance ratio (*P/T*), signal-to-noise ratio (*SNR*), discrimination ratio (*DR*), and process capability index (*Cp* or *Cpk*). Costa et al. (2005) addressed the design and implementation of a measurement system that permitted the evaluation—quantitatively, objectively, and systematically—of the superficial paper waviness in industrial practice. The process of designing the measurement system was presented considering all its stages, from selection and evaluation of the measuring device (using GR&R) to the generation and validation of the statistical model of measurement.

Lyu and Chen (2008) developed a procedure, based on the generalized linear model, to evaluate the repeatability and reproducibility of a measurement system for data-type attributes. To calculate the repeatability of the system, the procedure integrated the iterative weighted least squares (IWLS) method and deviance analysis. Senol (2004) used an experimental design including laboratory factors as a measurement variability factor in MSA studies. This study concluded that environmental and atmospheric conditions, often overlooked in GR&R studies, might represent a significant contribution to the variability in measurements. Shiau (2000) used a mathematical model of the measurement cost-loss for a measuring device as well as to enhance its use on-line. The study suggested that the guard limits concept can both reduce on-line measurement loss cost and be applied to adopting a marginal gage when no better gage is available.

1.2. Multivariate approach for GR&R studies

The bulk of the studies associated with analyzing the quality and efficiency of measurement systems are so far limited to a discussion of one single critical-to-quality characteristic (CTQ). Currently, the ANOVA method for GR&R studies can be applied only to univariate data (Wang and Yang, 2007). In assessing measurement systems that measure multiple characteristics, the analyst must consider the correlation structure of the CTQs, a task more suited to multivariate methods. Flynn et al. (2009) used regression analysis to analyze the comparative performance capability between two functionally equivalent but technologically different automatic measurement systems. The systems were used for acceptance testing of a unit under test. For such accurate measurements as repeatability and reproducibility, the "pass/fail" criteria for a test unit were inappropriate. Hence, the authors proposed a methodology that used principal component analysis (PCA) and multivariate analysis of variance (MANOVA) to examine whether there was a statistically significant difference between the system's measurements. He et al. (2011) proposed an online multivariate MSA approach to detecting faulty test instruments in a multisite testing system. The multivariate data were transformed using PCA. The values of the principal components of each test instrument were then compared with the control limits obtained by analyzing the principal components of all test instruments.

Majeske (2008) used the MANOVA method to estimate the variance-covariance matrix for GR&R studies with one, two, and three significant factors. This work evaluated a measurement system using data from a GR&R study of a sheet-metal body panel. In doing so it demonstrated how to adjust a MANOVA model and estimate multivariate criteria (P/T_m, $\%R\&R_m$ and SNR_m). Wang and Chien (2010) used the process-oriented basis representation method (POBREP) to evaluate a measurement process with multivariate data. The results showed that POBREP outperformed other methods such as PCA and ANOVA. The POBREP was able to identify specific causes of production problems and map those into a basis matrix. Wang and Yang (2007) presented a GR&R study with multiple characteristics using the PCA method. To assess the adequacy of the measurement system, the study employed two composite indexes: precision-to-tolerance ratio and measurement-variation-to-total-variation-of-measurement-system ratio. The case study showed that, for estimating the indexes, PCA outperformed the ANOVA method.

This article deals with a multivariate analysis of a measurement system through studies of repeatability and reproducibility of the measurement process. Its main objective is two-fold: to propose a new method for multivariate analysis of a measurement system and to assess the performance between the proposed method and those found in the literature. The new method, Weighted Principal Components (WPC), ponders the scores in principal component analysis by their eigenvalues. To prove its efficiency, the study evaluated measurements of roughness parameters, obtained from AISI 12L14 steel turning machined with carbide tools. In this GR&R study, the following are considered: 12 parts, 3 operators, 4 replicates, and 5 responses (R_a, R_y, R_z, R_q and R_t) with a correlation structure that determined 86.2% of explanation to the first principal component. In addition, simulated data are generated with different correlation structures and measurement systems that are unacceptable, marginal (may be acceptable depending on application), and acceptable. The results obtained by WPC are then compared to those obtained through methods found in the literature. The numerical example shows that ANOVA is not a suitable means of treating multiple responses with significant correlations. The simulation study concludes that the proposed multivariate method is more robust than that of MANOVA and PCA.

The remainder of this paper is structured as follows. Section 2 shows how to evaluate a measurement system using the ANOVA, MANOVA, and PCA methods. Section 3 details the WPC method proposed by the authors. Section 4 presents GR&R studies applied to the roughness parameters of AISI 12L14 steel turning; the data is evaluated using univariate and multivariate methods. In Section 5, a simulation study is conducted to evaluate the performance of the methods, especially the multivariate, for different correlation structures as well as for measurement systems that are unacceptable, marginal, and acceptable. Finally, Section 6 presents the main findings involving the analysis using the univariate method ANOVA and the multivariate methods MANOVA, PCA, and WPC.

2. Literature methods for GR&R study

2.1. Univariate GR&R study based on ANOVA

In many processes involving measurements of manufactured products for a single CTQ, the variability may be due to a measurement error, to variability in the measuring device, or to variability in the product/process itself. A complete model for a GR&R study with p parts, o operators, and r replicates is made up of a two-factor crossed design with interaction as such (Al-Refaie and Bata, 2010; Burdick et al., 2003; Deldossi and Zappa, 2011; Erdmann et al., 2010):

$$\text{ctq} = \mu+\alpha_i+\beta_j+(\alpha\beta)_{ij}+\varepsilon_{ijk}\begin{cases} i=1,2,\ldots,p \\ j=1,2,\ldots,o \\ k=1,2,\ldots,r \end{cases} \tag{1}$$

where ctq is the response variable measured; μ is the mean of the measured values; $\alpha_i \sim N(0,\sigma_\alpha)$, $\beta_j \sim N(0,\sigma_\beta)$, $\alpha\beta_{ij} \sim N(0,\sigma_{\alpha\beta})$and $\varepsilon_{ijk} \sim N(0,\sigma_\varepsilon)$ are random and statistically independent variables of parts, operators, interaction and the error term, respectively. The above components of variance can be translated into notation GR&R to (Kaija et al., 2010; Li and Al-Refaie, 2008; Senol, 2004; White and Borror, 2011):

$$\sigma_P^2=\sigma_\alpha^2,\quad \sigma_{\text{repeatability}}^2=\sigma_\varepsilon^2,\quad \sigma_{\text{reproducibility}}^2=\sigma_\beta^2+\sigma_{\alpha\beta}^2$$
$$\sigma_{MS}^2=\sigma_{\text{repeatability}}^2+\sigma_{\text{reproducibility}}^2,\quad \sigma_T^2=\sigma_P^2+\sigma_{MS}^2 \tag{2}$$

The variance components of Model (1) in Eq. (2) can be estimated using the method Analysis of Variance (ANOVA). More details on how to calculate the components of variation using ANOVA can be found in Majeske (2008) and Wang and Chien (2010).

To determine the acceptability of a measurement system, analysts commonly uses two indexes in GR&R studies. The AIAG (2010) recommended evaluating a measurement system by scaling the standard deviation of measurement error to the total standard deviation of the observed process. This statistic, called the percentage of *R&R*, is defined as

$$\%R\&R=\left(\frac{\sigma_{MS}}{\sigma_T}\right)100\% \tag{3}$$

If the measurement system is, according to the index, less than 10%, it is considered acceptable. If between 10% and 30%, it is considered marginal—acceptable depending on the application, the cost of the measurement device, the cost of repair and other factors. If it exceeds 30%, it is considered unacceptable and should be improved (AIAG, 2010; Al-Refaie and Bata, 2010; Montgomery, 2005; Woodall and Borror, 2008).

The number of distinct categories (*ndc* or signal to noise ratio, *SNR*) is defined in Eq. (4). A value of five or greater is recommended; a value less than two indicates that the measurement system is unable to monitor the process (AIAG, 2010; Burdick et al., 2003; Li and Al-Refaie, 2008).

$$ndc=\sqrt{\frac{2\sigma_P^2}{\sigma_{MS}^2}}=\sqrt{2}\,\frac{\sigma_P}{\sigma_{MS}} \tag{4}$$

2.2. Multivariate GR&R study based on MANOVA

When a GR&R study considers a two-factors cross design with interaction for multiple CTQs (q characteristics), the model is given as (He et al., 2011; Majeske, 2008; Wang and Chien, 2010):

$$\mathbf{CTQ}=\begin{bmatrix} \mathrm{CTQ}_{11} & \mathrm{CTQ}_{12} & \cdots & \mathrm{CTQ}_{1q} \\ \mathrm{CTQ}_{21} & \mathrm{CTQ}_{22} & \cdots & \mathrm{CTQ}_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{CTQ}_{n1} & \mathrm{CTQ}_{n2} & \cdots & \mathrm{CTQ}_{nq} \end{bmatrix}$$
$$=\boldsymbol{\mu}+\boldsymbol{\alpha}_\mathbf{i}+\boldsymbol{\beta}_\mathbf{j}+(\boldsymbol{\alpha\beta})_\mathbf{ij}+\boldsymbol{\varepsilon}_\mathbf{ijk}\begin{cases} i=1,2,\ldots,p \\ j=1,2,\ldots,o \\ k=1,2,\ldots,r \end{cases} \tag{5}$$

where $\mathbf{CTQ}=(\mathbf{CTQ_1}, \mathbf{CTQ_2},\ldots,\mathbf{CTQ_q})$ and $\boldsymbol{\mu}=(\boldsymbol{\mu}_\mathbf{1}, \boldsymbol{\mu}_\mathbf{2},\ldots,\boldsymbol{\mu}_\mathbf{q})$ are constant vectors; $\boldsymbol{\alpha}_\mathbf{i}\sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma}_\alpha)$, $\boldsymbol{\beta}_\mathbf{j}\sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma}_\beta)$, $\boldsymbol{\alpha\beta}_\mathbf{ij}\sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma}_{\alpha\beta})$, and $\boldsymbol{\varepsilon}_\mathbf{ijk}\sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma}_\varepsilon)$ are random vectors statistically independent of each other. The above components of variance can be translated into notation GR&R to

$$\hat{\boldsymbol{\Sigma}}_\mathbf{P}=\hat{\boldsymbol{\Sigma}}_\alpha,\quad \hat{\boldsymbol{\Sigma}}_\mathbf{repeatability}=\hat{\boldsymbol{\Sigma}}_\varepsilon,\quad \hat{\boldsymbol{\Sigma}}_\mathbf{reproducibility}=\hat{\boldsymbol{\Sigma}}_\beta+\hat{\boldsymbol{\Sigma}}_{\alpha\beta}$$
$$\hat{\boldsymbol{\Sigma}}_\mathbf{MS}=\hat{\boldsymbol{\Sigma}}_\mathbf{repeatability}+\hat{\boldsymbol{\Sigma}}_\mathbf{reproducibility},\quad \hat{\boldsymbol{\Sigma}}_\mathbf{T}=\hat{\boldsymbol{\Sigma}}_\mathbf{P}+\hat{\boldsymbol{\Sigma}}_\mathbf{MS} \tag{6}$$

The variance components of Model (5) in Eq. (6) can be estimated using the method of Multivariate Analysis of Variance (MANOVA). Before estimating the variance-covariance matrices, $\boldsymbol{\Sigma}_\mathbf{p}$, $\boldsymbol{\Sigma}_\mathbf{MS}$ and $\boldsymbol{\Sigma}_\mathbf{T}$ are calculated as mean squares matrices for part, operator, part*operator interaction and the error term. More details on how to calculate these components of variation using MANOVA for multivariate GR&R studies can be found in Majeske (2008).

The multivariate indexes $\%R\&R_m$ and ndc_m are calculated by Eqs. (7) and (8), respectively. λ_{P_i},λ_{MS_i} and λ_{T_i} $i=1,2,\ldots,q$ are eigenvalues extracted from variance-covariance matrices, $\mathbf{\Sigma_p}$, $\mathbf{\Sigma_{MS}}$ and $\mathbf{\Sigma_{pT}}$. The acceptance criteria for the measurement system for the multivariate indexes $\%R\&R_m$ and ndc_m are the same for univariate indexes *%R&R* and *ndc* (Majeske, 2008).

$$\%R\&R_m = \left(\prod_{i=1}^{q}\sqrt{\frac{\boldsymbol{\lambda}_{\mathbf{MS_i}}}{\boldsymbol{\lambda}_{\mathbf{T_i}}}}\right)^{1/q} 100\% \tag{7}$$

$$ndc_m = \sqrt{2}\left(\prod_{i=1}^{q}\sqrt{\frac{\boldsymbol{\lambda}_{\mathbf{P_i}}}{\boldsymbol{\lambda}_{\mathbf{MS_i}}}}\right)^{1/q} \tag{8}$$

2.3. Multivariate GR&R study based on PCA

In dealing with multiple CTQs in a GR&R study, an alternative method to Majeske's (2008) MANOVA is, according to Wang and Chien (2010), PCA. PCA is one of the most widely applied tools used to summarize the common patterns of variation among variables. Furthermore, this statistical technique is able to retain significant information from the first axes of the PCs, since the variations associated with experimental error, measurement error, rounding error are summarized in the last axes of PCs (Paiva et al., 2007).

PCA is algebraically a linear combination ℓ of q random variables $\mathrm{CTQ_1,CTQ_2,\ldots,CTQ_q}$. Geometrically these combinations represent a new coordinate system obtained during the rotation of an original system (Johnson and Wichern, 2002; Mukherjee and Ray, 2008; Paiva et al., 2008). The coordinates of the axes now have the variables $\mathrm{CTQ_1,CTQ_2,\ldots,CTQ_q}$ and represent the direction of the maximum. The principal components are uncorrelated and depend only on the variance-covariance matrix $\mathbf{\Sigma}$ (or the correlation matrix $\mathbf{R}$) of original variables and their development does not require the assumption of multivariate normality. The *i*th principal component can be obtained according to Eqs. (9) and (10).

$$\begin{aligned} &Maximize: \quad Var\left[\mathbf{e_i^T CTQ}\right] \\ &\quad Subject\ \ to: \quad \mathbf{e_i^T e_i}=1 \\ &\quad Cov\left[\mathbf{e_i^T CTQ},\mathbf{e_k^T CTQ}\right]=0, \quad k<i \end{aligned} \tag{9}$$

$$\mathrm{PC_i} = e_i^T\mathrm{CTQ} = \mathbf{e_{1i}}\mathrm{CTQ_1}+\mathbf{e_{2i}}\mathrm{CTQ_2}+\ldots+\mathbf{e_{qi}}\mathrm{CTQ_q} \quad i=1,2,\ldots,q \tag{10}$$

The lexicographical solution to the multiobjective program in Eq. (9) provides pairs of eigenvalues–eigenvectors $(\lambda_1,\mathbf{e_1})$, $(\lambda_2,\mathbf{e_2}),\ldots,(\lambda_q,\mathbf{e_q})$, where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_q \geq 0$, to obtain both percentage of explanation for each principal component and component scores using Eq. (10). Eq. (11) represents a complete model for a multivariate GR&R study with q quality characteristics, p parts, o operators, and r replicates, a model that can be analyzed by PCA. This model is similar to a univariate model. The original responses, however, are replaced by the principal component scores.

$$\mathrm{PC_q} = \mu+\alpha_i+\beta_j+(\alpha\beta)_{ij}+\varepsilon_{ijk} \begin{cases} i=1,2,\ldots,p \\ j=1,2,\ldots,o \\ k=1,2,\ldots,r \end{cases} \tag{11}$$

The variable μ is a constant and α_i, β_j, $(\alpha\beta)_{ij}$, ε_{ijk} are independent normal random variables with zero mean and variance, σ^2_α, σ^2_β, $\sigma^2_{\alpha\beta}$, and σ^2_ε, for parts (process), operators, part*operator interaction, and error term, respectively. As with the univariate approach, these components of variance can, according to Eq. (2), be translated into GR&R notation. Multivariate indexes for assessing the measurement system are estimated using Eqs. (3) and (4). Note that all calculations here are obtained based on scores of principal components.

3. Multivariate GR&R study based on weighted principal components (WPC)

In their analysis of measurement systems, Wang and Chien (2010) compared the PCA to two other methods. However, the authors conducted the analysis separately for each principal component. This methodology may be inappropriate; analyzing each component individually may provide different interpretations. When responses have high correlations ($\mathrm{PC_1}\% > 95\%$), analysis of the first principal component explains properly the variability of the measurement system. When correlations between the responses are not high, however, it becomes necessary to analyze more than one principal component. Indeed, the first principal component alone cannot explain the whole data set.

Therefore, this article proposes the method of a multivariate GR&R study using weighted principal components (WPC). In this case, the model response is the result of weighting principal component scores by their respective eigenvalues. This proposal is based on the work of Paiva et al. (2010), who used a technique of multi-objective optimization based on a weighting of the principal components. They used the technique to study a welding process with a multiple set of moderately correlated responses. The WPC method is schematically detailed in Fig. 1 and its steps are described in the following subsections.

3.1. Step 1: assess correlation between CTQs

First, the correlation between CTQs can be obtained by

$$Corr_{\mathrm{CTQ}_i\mathrm{CTQ}_j} = \frac{\mathrm{Covar}_{CTQ_iCTQ_j}}{\sqrt{\mathrm{Var}_{\mathrm{CTQ}_i}\mathrm{Var}_{\mathrm{CTQ}_j}}} \quad \forall i=1,2,\ldots,q;\ j=1,2,\ldots,q \tag{12}$$

where $\mathrm{Var}_{\mathrm{CTQ}_i}$ and $\mathrm{Var}_{\mathrm{CTQ}_j}$ are *i*th and *j*th variances; $\mathrm{Covar}_{\mathrm{CTQ}_i\mathrm{CTQ}_j}$ represents the covariance between CTQs.

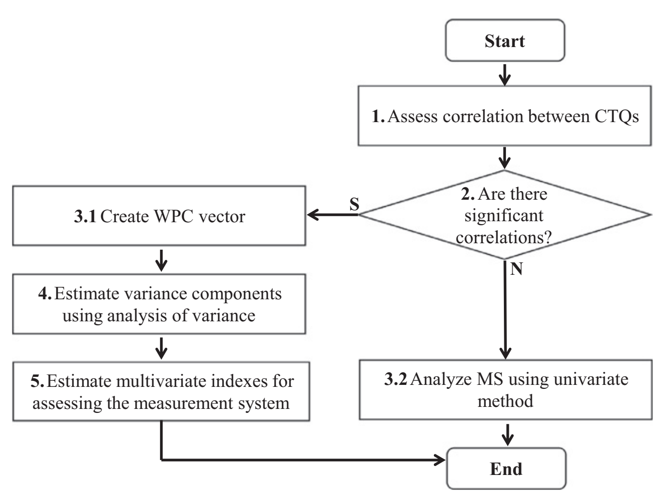

Fig. 1. Detailed flowchart for conducting the WPC method.

3.2. Step 2: verify whether significant correlations exist

If significant correlations exist between CTQs, proceed to step 3.1 for a multivariate GR&R study, which will start the process of evaluating the measurement system using the WPC method. If such correlations are absent, proceed to step 3.2 to use the univariate classic method, ANOVA.

3.3. Step 3.1: create WPC vector

The ith principal component can be obtained according to Eqs. (9) and (10). The lexicographical solution to the multiobjective program in Eq. (9) provides pairs of eigenvalues–eigenvectors $(\lambda_1,\mathbf{e_1}),(\lambda_2,\mathbf{e_2}),\ldots,(\lambda_q,\mathbf{e_q})$, where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_q \geq 0$, to obtain both percentage of explanation for each principal component and principal component scores. Considering the matrix of standardized data ($\mathbf{Z}$) and the matrices of eigenvalues ($\mathbf{L}$) and eigenvectors ($\mathbf{E}$), the weighted scores of principal components are obtained from Eq. (13).

$$\mathbf{WPC}=\mathbf{Z}^{\mathbf{T}}\mathbf{E}\ \ \mathbf{L}=\begin{bmatrix}\left(\frac{CTQ_{11}-\overline{CTQ}_1}{\sqrt{s_{11}}}\right) & \left(\frac{CTQ_{12}-\overline{CTQ}_2}{\sqrt{s_{22}}}\right) & \cdots & \left(\frac{CTQ_{1q}-\overline{CTQ}_q}{\sqrt{s_{qq}}}\right)\\ \left(\frac{CTQ_{21}-\overline{CTQ}_1}{\sqrt{s_{11}}}\right) & \left(\frac{CTQ_{22}-\overline{CTQ}_2}{\sqrt{s_{22}}}\right) & \cdots & \left(\frac{CTQ_{2q}-\overline{CTQ}_q}{\sqrt{s_{qq}}}\right)\\ \vdots & \vdots & \ddots & \vdots\\ \left(\frac{CTQ_{n1}-\overline{CTQ}_1}{\sqrt{s_{11}}}\right) & \left(\frac{CTQ_{n2}-\overline{CTQ}_2}{\sqrt{s_{22}}}\right) & \cdots & \left(\frac{CTQ_{nq}-\overline{CTQ}_q}{\sqrt{s_{qq}}}\right)\end{bmatrix}$$

$$\times\begin{bmatrix}e_{11} & e_{12} & \cdots & e_{1q}\\ e_{21} & e_{22} & \cdots & e_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ e_{q1} & e_{q2} & \ldots & e_{qq}\end{bmatrix}\times\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_q\end{bmatrix} \tag{13}$$

3.4. Step 4: estimate variance components

The proposed model to evaluate a measurement system using multivariate GR&R study is given by

$$\mathbf{WPC}=\mu+\alpha_i+\beta_j+(\alpha\beta)_{ij}+\varepsilon_{ijk}\begin{cases} i=1,2,\ldots,p\\ j=1,2,\ldots,o\\ k=1,2,\ldots,r\end{cases} \tag{14}$$

that is, the response used in Model (14) is the result of weighting the principal components by their eigenvalues, according to Eq. (13). The variable μ is a constant and α_i, β_j, $\alpha\beta_{ij}$and ε_{ijk} are independent normal random variables with zero mean and variance, σ^2_α, σ^2_β, $\sigma^2_{\alpha\beta}$, and σ^2_ε, respectively. Johnson and Wichern (2002) provide a variety of rules to estimate the appropriate number of non-trivial PCA axes (PC scores) that can be taken to represent the original data set. However, due to the weighting of the principal components by their eigenvalues, all principal components can be included in the model. The components with eigenvalues of greater importance are weighted more, and all the information is included in the study. The components of variance in Model (14), based on transformation of the original data-set into WPC, can be translated into GR&R notation by

$$\hat{\sigma}^2_P=\hat{\sigma}^2_\alpha=\frac{MSP-MSPO}{or} \tag{15}$$

$$\hat{\sigma}^2_{\text{reproducibility}}=\hat{\sigma}^2_\beta+\hat{\sigma}^2_{\alpha\beta}=\frac{MSO-MSPO}{pr}+\frac{MSPO-MSE}{r} \tag{16}$$

$$\hat{\sigma}^2_{\text{repeatability}}=\hat{\sigma}^2_\varepsilon=MSE \tag{17}$$

$$\hat{\sigma}^2_{MS}=\hat{\sigma}^2_{\text{repeatability}}+\hat{\sigma}^2_{\text{reproducibility}} \tag{18}$$

$$\hat{\sigma}^2_T=\hat{\sigma}^2_P+\hat{\sigma}^2_{MS} \tag{19}$$

where MSP, MSO, $MSPO$, and MSE are, respectively, the mean squares for the factors part, operator, interaction, and the error term. Also, if the interaction effect is not significant, the complete model can be reduced to

$$\mathbf{WPC}=\mu+\alpha_i+\beta_j+\varepsilon_{ijk} \tag{20}$$

Now the components of variance for parts (process) and reproducibility (operators) in Model (20) are estimated by

$$\hat{\sigma}^2_P=\hat{\sigma}^2_\alpha=\frac{MSP-MSE}{or} \tag{21}$$

$$\hat{\sigma}^2_{\text{reproducibility}}=\hat{\sigma}^2_\beta=\frac{MSO-MSE}{pr} \tag{22}$$

3.5. Step 5: estimate multivariate indexes for assessing the measurement system

After variance components have been calculated for a multivariate GR&R study, $\%R\&R_m$ and ndc_m indexes can be estimated by using Eqs. (23) and (24). The acceptance criteria for classifying the measurement system using multivariate indexes, $\%R\&R_m$ and ndc_m, are the same for the univariate indexes, $\%R\&R$ and ndc.

$$\%R\&R_m=\left(\frac{\sigma_{MS}}{\sigma_T}\right)100\% \tag{23}$$

$$ndc_m=\sqrt{\frac{2\sigma^2_P}{\sigma^2_{MS}}}=\sqrt{2}\frac{\sigma_P}{\sigma_{MS}} \tag{24}$$

Despite the fact that Eqs. (23) and (24) bear a similarity to Eqs. (3) and (4), all calculations here have been obtained based on weighted scores of principal components. In fact, correlated CTQs were transformed into uncorrelated scores of principal components and those scores were thereby weighted by their eigenvalues for dimensionality reduction in multivariate GR&R studies.

4. Numerical example

In this multivariate GR&R study, five roughness parameters are analyzed: R_a (arithmetic average), R_y (maximum), R_z (ten point height), R_q (root mean square), and R_t (maximum peak to valley).

Fig. 2. AISI 12L14 steel machined on a CNC lathe with a 5.5 kW spindle motor with conventional roller bearings.

The work piece, machined on a CNC lathe (see Fig. 2), was AISI 12L14 steel (0.090% C, 0.030% Si, 1.240% Mn, 0.046% P, 0.273% S, 0.150% Cr, 0.080% Ni, 0.260% Cu, 0.001% Al, 0.020% Mo, 0.280% Pb, 0.0079% N2). The machining parameters used in this study were cutting speed of 345 m min^{-1}, feed rate of 0.086 mm rev^{-1}, and depth of cut of 0.680 mm. Carbide inserts were used of ISO P35 class, coated with three toppings (Ti (C, N), Al_2O_3, TiN), (GC Sandvik 4035) geometry ISO SNMG 09 03 04 – PM, and tool holder ISO DSBNL 1616H 09.

Using these experimental conditions, four parts were machined and three noise conditions considered: slenderness of the part, measuring position, and tool wear. The slenderness (S) relates the diameter (D) with the length (L) of the part, according to the relation $S=L/D$. The parts were classified as slender and non-slender, for the same part length, with $D=30$ mm and $D=50$ mm, respectively. The following regions of measurement were selected: close to the main spindle, center, and close to the barrel. The tool wear noise considered new tool and worn tool which it was measured on the edge of approximately 0.3 mm.

Using experimental design and considering the noise conditions mentioned above, the GR&R study adopted $p=12$ parts, $o=3$ operators, and $r=4$ replicates. Table 1 contains the data for the GR&R study. The device evaluated in this study was a portable roughness meter, shown in Fig. 3, set to a cut-off length of 0.25.

4.1. ANOVA

A strategy commonly used in quality improvement projects is to prioritize the quality characteristic. For this study the parameter R_a, widely used in most manufacturing processes, was selected to evaluate the measurement system. To run the univariate GR&R study, a two-way analysis of variance with interaction, Eq. (1), was adjusted to R_a. The interaction term was not significant for a significance level of 0.05, so the model could be adjusted to a reduced model without the interaction term. Then, variances in Eq. (2) were estimated and their square roots appear in Table 2 as $\hat{\sigma}_P$, $\hat{\sigma}_{MS}$ and $\hat{\sigma}_T$. Eqs. (3) and (4) were used to estimate the %R&R and *ndc* indexes. The index %R&R = 18.62% classifies the

Table 1
Observed measurements of roughness parameters for the multivariate GR&R study.

		$j=1$					$j=2$					$j=3$				
i	k	R_z	R_y	R_t	R_q	R_a	R_z	R_y	R_t	R_q	R_a	R_z	R_y	R_t	R_q	R_a
1	1	6.34	7.73	8.81	1.67	1.39	6.37	8.29	9.25	1.67	1.38	6.39	8.38	9.25	1.67	1.38
2	1	7.61	9.19	9.46	1.85	1.54	7.57	9.17	9.43	1.84	1.53	7.60	9.18	9.41	1.84	1.53
3	1	6.43	7.67	7.81	1.50	1.23	6.11	7.78	8.46	1.55	1.27	6.44	7.69	7.83	1.50	1.23
4	1	7.52	9.01	9.01	2.10	1.84	7.53	8.86	8.94	2.10	1.84	7.54	9.03	9.03	2.10	1.84
5	1	7.28	7.77	8.04	2.30	2.07	7.26	7.84	8.02	2.27	2.05	7.26	7.84	8.02	2.27	2.05
6	1	7.34	7.92	7.94	2.35	2.12	7.33	7.92	7.95	2.37	2.14	7.32	7.94	7.96	2.37	2.14
7	1	9.24	10.55	10.55	2.39	1.93	9.27	10.55	10.55	2.39	1.92	9.22	10.54	10.54	2.38	1.91
8	1	3.74	4.04	4.04	0.93	0.75	3.95	4.32	4.32	0.96	0.77	3.88	4.32	4.32	0.97	0.78
9	1	4.17	4.84	5.02	0.98	0.79	4.27	4.87	5.02	1.02	0.81	4.24	4.89	5.05	0.99	0.79
10	1	7.70	8.62	9.25	2.04	1.70	7.83	8.67	9.16	2.10	1.76	7.79	8.66	9.11	2.10	1.76
11	1	6.85	7.51	7.51	2.01	1.73	6.83	7.49	7.53	2.00	1.72	6.83	7.46	7.54	2.00	1.72
12	1	7.45	8.29	8.29	2.21	1.95	7.47	8.24	8.24	2.21	1.95	7.46	8.22	8.22	2.21	1.94
1	2	6.63	7.56	7.79	1.80	1.52	6.64	7.57	7.78	1.80	1.52	6.63	7.58	7.74	1.80	1.52
2	2	6.46	8.75	8.85	1.58	1.20	6.03	9.55	9.55	1.47	1.17	5.96	9.30	9.30	1.46	1.17
3	2	6.81	8.27	8.76	1.65	1.29	6.72	7.85	8.82	1.56	1.19	6.86	8.13	8.77	1.61	1.24
4	2	8.02	9.71	10.12	2.04	1.75	8.10	9.78	10.26	2.05	1.76	8.05	9.70	10.22	2.05	1.76
5	2	7.00	7.43	7.58	2.29	2.08	7.00	7.40	7.61	2.29	2.08	7.01	7.42	7.63	2.29	2.08
6	2	7.31	7.82	7.82	2.38	2.15	7.32	7.78	7.92	2.40	2.18	7.34	7.81	7.96	2.38	2.16
7	2	8.80	10.26	10.26	2.31	1.85	8.72	9.43	9.63	2.34	1.89	8.67	9.64	9.85	2.28	1.82
8	2	4.02	4.77	4.77	1.02	0.82	4.07	4.58	4.58	1.02	0.83	3.91	4.27	4.54	1.00	0.81
9	2	3.92	4.26	4.53	0.99	0.80	3.89	4.26	4.51	0.99	0.80	3.90	4.24	4.49	0.99	0.80
10	2	7.75	9.24	9.91	2.06	1.72	7.79	9.22	9.93	2.07	1.73	7.78	9.23	9.91	2.07	1.72
11	2	6.65	7.23	7.29	2.01	1.73	6.64	7.20	7.39	2.01	1.74	6.64	7.19	7.30	2.01	1.73
12	2	7.53	8.48	8.48	2.17	1.88	7.46	8.47	8.47	2.17	1.89	7.45	8.21	8.25	2.17	1.88
1	3	6.05	7.03	7.07	1.52	1.26	6.06	7.10	7.10	1.51	1.26	6.05	7.00	7.12	1.51	1.26
2	3	5.70	7.60	7.60	1.37	1.11	5.67	7.51	7.51	1.39	1.12	5.67	7.56	7.56	1.37	1.10
3	3	6.51	8.46	8.58	1.65	1.37	5.57	9.18	9.18	1.33	1.03	6.41	8.94	8.94	1.70	1.43
4	3	7.59	9.62	9.62	2.00	1.71	7.65	10.07	10.07	2.03	1.73	7.66	9.86	9.86	2.02	1.72
5	3	7.07	7.45	7.65	2.26	2.04	7.09	7.43	7.65	2.26	2.04	7.09	7.46	7.68	2.26	2.03
6	3	7.19	7.89	7.89	2.39	2.17	7.18	7.86	7.86	2.37	2.16	7.15	7.76	7.76	2.37	2.16
7	3	9.46	10.16	10.23	2.43	1.96	9.54	10.42	10.46	2.44	1.97	9.53	10.36	10.40	2.44	1.96
8	3	3.80	3.92	4.04	1.00	0.81	3.79	3.96	4.06	0.96	0.78	3.75	3.92	4.00	0.95	0.77
9	3	3.90	4.00	4.06	1.04	0.83	3.81	3.89	4.03	1.01	0.80	3.78	3.95	4.03	0.99	0.79
10	3	8.35	10.08	11.38	2.10	1.71	8.49	10.06	11.47	2.11	1.72	8.71	10.22	11.82	2.13	1.72
11	3	6.77	7.10	7.22	2.04	1.76	6.76	7.19	7.23	2.03	1.75	6.76	7.15	7.21	2.03	1.75
12	3	7.70	8.02	8.42	2.22	1.93	7.69	8.01	8.40	2.23	1.94	7.70	8.02	8.40	2.23	1.94
1	4	6.39	7.47	7.55	1.64	1.35	6.32	7.47	7.47	1.63	1.34	6.34	7.49	7.49	1.64	1.35
2	4	7.86	9.40	9.77	1.86	1.50	7.85	9.47	9.83	1.86	1.49	7.93	9.70	10.16	1.86	1.49
3	4	7.00	8.79	8.79	1.75	1.42	7.06	8.81	8.92	1.76	1.42	6.94	8.60	8.85	1.75	1.42
4	4	8.07	10.18	10.18	2.18	1.89	8.04	9.49	9.49	2.18	1.89	8.03	9.49	9.56	2.19	1.90
5	4	6.96	7.16	7.30	2.28	2.07	6.97	7.28	7.38	2.28	2.07	6.98	7.29	7.40	2.28	2.07
6	4	7.24	7.56	7.74	2.35	2.12	7.25	7.49	7.75	2.35	2.12	7.25	7.41	7.70	2.36	2.13
7	4	8.10	8.56	8.73	2.18	1.78	8.11	8.59	8.76	2.18	1.78	8.10	8.54	8.69	2.17	1.77
8	4	4.34	5.36	5.36	1.09	0.88	4.15	5.08	5.14	1.06	0.85	4.07	5.14	5.21	1.03	0.83
9	4	4.49	4.74	5.11	1.12	0.90	4.83	5.90	6.42	1.17	0.94	4.78	6.73	6.78	1.18	0.94
10	4	7.36	8.54	8.99	1.98	1.68	7.39	8.71	9.03	1.99	1.68	7.40	8.76	9.06	1.99	1.68
11	4	6.73	7.13	7.28	2.01	1.73	6.69	7.08	7.23	2.00	1.72	6.68	7.07	7.20	1.99	1.71
12	4	7.54	8.09	8.43	2.08	1.79	7.58	8.12	8.29	2.09	1.80	7.56	8.17	8.29	2.11	1.81

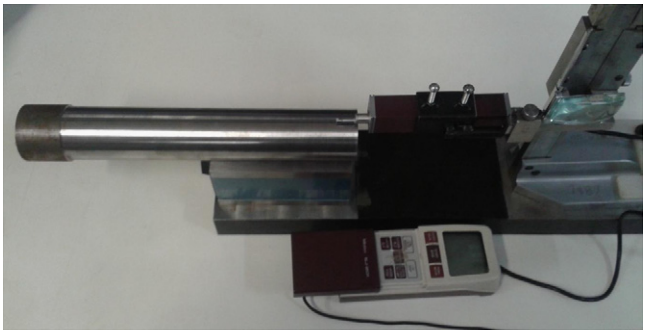

Fig. 3. Mitutoyo portable roughness meter model Surftest SJ-201P.

Table 2
Roughness meter classification through ANOVA, PCA, and WPC methods.

	ANOVA					PCA		
	R_a	R_y	R_z	R_q	R_t	PC_1	PC_2	WPC
$\hat{\sigma}_P$	0.443	1.689	1.431	0.469	1.744	2.0907	0.7792	9.0054
$\hat{\sigma}_{MS}$	0.084	0.544	0.407	0.095	0.634	0.5302	0.2808	2.3764
$\hat{\sigma}_T$	0.451	1.774	1.488	0.479	1.856	2.1569	0.8282	9.3137
%R&R	18.62%	30.66%	27.37%	19.79%	34.14%	24.58%	33.91%	25.52%
ndc	7	4	4	6	3	5	3	5

Table 3
Correlation structure between roughness parameters.

	R_z	R_y	R_t	R_q
R_y	0.920[a]	–	–	–
	0.000[b]	–	–	–
R_t	0.908	0.988	–	–
	0.000	0.000	–	–
R_q	0.906	0.734	0.708	–
	0.000	0.000	0.000	–
R_a	0.839	0.652	0.623	0.989
	0.000	0.000	0.000	0.000

[a] Pearson correlation.
[b] *p*-value.

measurement system as marginal, having potential for improvement. The index $ndc=7$, being more than 5, classifies the measuring system as acceptable.

However, the prioritization of the CTQs is not satisfactory in evaluating the surface roughness of a machined part. The parameter R_a alone is insufficient for describing a surface completely. Its disadvantage is evident when a non-typical peak or valley is detected on the surface. Nevertheless, it does not interfere in the calculation of the average value, keeping the defect nearly concealed. To evaluate R_a, the analyst may be interested in a complementary parameter. The parameter R_y has wide acceptance and can be a good choice because it provides information about the deterioration of the vertical surface part.

To run the univariate GR&R study, a two-way analysis of variance with interaction, Eq. (1), was adjusted to R_y. The interaction term was not significant for a significance level of 0.05, so the model could be adjusted to a reduced model without the interaction term. Then, variances in Eq. (2) were estimated and their square roots appear in Table 2 as $\hat{\sigma}_P$, $\hat{\sigma}_{MS}$ and $\hat{\sigma}_T$. Finally, Eqs. (3) and (4) were used to estimate the *%R&R* and *ndc* indexes. The index *%R&R*=30.66% classified the measurement system as unacceptable. The index $ndc=4$ states that the measurement system was able to identify only four distinct categories of parts.

Individually, the parameter R_y also fails to provide sufficient information about the surface. Indeed, various forms of surface roughness may have the same value R_y. When R_a and R_y (as a supplement to R_a) are evaluated using univariate statistical techniques, the analyst cannot verify the measurement system's acceptability. Since the same measurement device measures all roughness parameters, a more detailed analysis was able to show that these responses were highly correlated (see Table 3). Therefore, assessing the measurement system considering independent responses may not be the most appropriate method. It was decided to display the parameters of surface roughness in a vector and use a multivariate approach to evaluate the measurement system.

4.2. MANOVA

Before beginning the measurement system analysis using MANOVA, the data in Table 1 were standardized by subtracting the mean and dividing by the standard deviation for each observation. The standardization of data is important not only when the variables are in different units but also when the variables are at different scales. Thus, to perform this multivariate GR&R study, the analyst adjusted the standardized data of Table 1 using a two-way multivariate analysis of variance according to the model in Eq. (5). The interaction term was not significant for a significance level of 0.05. Thus, the model could be adjusted to a reduced model without the interaction term. Then, the matrices of the mean squares for the factor part, the factor operator, and the error term were estimated. The matrices are

$$\mathbf{MSP}=\begin{pmatrix} 12.5745 & 7.5039 & 10.1185 & 12.3004 & 7.2875 \\ 7.5039 & 11.1506 & 10.5769 & 8.2761 & 11.2526 \\ 10.1185 & 10.5769 & 11.8887 & 10.7970 & 10.5188 \\ 12.3004 & 8.2761 & 10.7970 & 12.2838 & 8.0766 \\ 7.2875 & 11.2526 & 10.5188 & 8.0766 & 11.4015 \end{pmatrix} \tag{25}$$

$$\mathbf{MSO}=\begin{pmatrix} 0.0070 & -0.0079 & 0.0090 & 0.0092 & 0.0063 \\ -0.0079 & 0.0088 & -0.0100 & -0.0104 & -0.0072 \\ 0.0090 & -0.0100 & 0.0117 & 0.0116 & 0.0069 \\ 0.0092 & -0.0104 & 0.0116 & 0.0123 & 0.0092 \\ 0.0063 & -0.0072 & 0.0069 & 0.0092 & 0.0109 \end{pmatrix} \tag{26}$$

$$\mathbf{MSE}=\begin{pmatrix} 0.0359 & 0.0479 & 0.0447 & 0.0424 & 0.0447 \\ 0.0479 & 0.1564 & 0.1017 & 0.0715 & 0.1353 \\ 0.0447 & 0.1017 & 0.0939 & 0.0664 & 0.0910 \\ 0.0424 & 0.0715 & 0.0664 & 0.0604 & 0.0658 \\ 0.0447 & 0.1353 & 0.0910 & 0.0658 & 0.1351 \end{pmatrix} \tag{27}$$

The variance-covariance matrices for the part, measurement system, and the total variation were then estimated. The result is shown here

$$\hat{\mathbf{\Sigma}}_{\mathbf{P}}=\begin{pmatrix} 1.0448 & 0.62133 & 0.83948 & 1.02150 & 0.60357 \\ 0.62133 & 0.91618 & 0.87293 & 0.68372 & 0.92643 \\ 0.83948 & 0.87293 & 0.98290 & 0.89422 & 0.86898 \\ 1.02150 & 0.68372 & 0.89422 & 1.01861 & 0.66757 \\ 0.60357 & 0.92643 & 0.86898 & 0.66757 & 0.93886 \end{pmatrix} \tag{28}$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{SM}} = \begin{pmatrix} 0.03530 & 0.04674 & 0.04397 & 0.04170 & 0.04387 \\ 0.04674 & 0.15328 & 0.09938 & 0.06981 & 0.13238 \\ 0.04397 & 0.09938 & 0.9214 & 0.06526 & 0.08922 \\ 0.04170 & 0.06981 & 0.06526 & 0.05941 & 0.06464 \\ 0.04387 & 0.13238 & 0.08922 & 0.06464 & 0.13251 \end{pmatrix} \quad (29)$$

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{T}} = \begin{pmatrix} 1.08018 & 0.66808 & 0.88345 & 1.06320 & 0.64744 \\ 0.66808 & 1.06947 & 0.97231 & 0.75353 & 1.05881 \\ 0.88345 & 0.97231 & 1.07505 & 0.95948 & 0.95820 \\ 1.06320 & 0.75353 & 0.95948 & 1.07803 & 0.73221 \\ 0.64744 & 1.05881 & 0.95820 & 0.73221 & 1.07137 \end{pmatrix} \quad (30)$$

Finally, to evaluate the multivariate indexes of the measuring system's acceptability, the eigenvalues of the matrices, $\hat{\boldsymbol{\Sigma}}_{\mathbf{P}}$, $\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{T}}$, were computed (Table 4). Using Eq. (7) the index $\%R\&R_m = 38.72\%$ was obtained. This represents one of the multivariate criteria used to assess the measurement system for the five CTQs R_a, R_y, R_z, R_q, and R_t. The fact that $\%R\&R_m > 30\%$ classifies the measurement system as unacceptable. Eq. (8) was used to calculate the multivariate index, $ndc_m = 3$. That is, the measurement system could distinguish only three categories of parts.

4.3. PCA

First, through the PCA method, an analysis was made of the principal components of responses R_a, R_y, R_z, R_q, and R_t using the correlation matrix of the data. The eigenvalues and eigenvectors obtained from the correlation matrix are presented in Table 5. Eqs. (9) and (10) were used to come up with the principal component scores. Wang and Chien (2010) evaluated only principal components with a cumulative percentage of explanation of at least 95% for the original variables. As the first principal component (PC_1) represents only 86.2% of the variability in the study, the scores of the second principal component (PC_2) were also analyzed. Taken together their sum accounted for 99.0% of the variability of the studied phenomenon. To perform a multivariate GR&R study, the PC_1 and PC_2 scores were adjusted by using two-way analysis of variance, according to the model in Eq. (11).

The study calculated the variances of parts, repeatability, reproducibility (operators), measurement system, and total variance. Table 2 presents the square roots of these variances and, with Eqs. (3) and (4), the $\%R\&R_m$ and ndc_m indexes for PC_1 and PC_2. For PC_1 and PC_2, the indexes ($\%R\&R_m$, ndc_m) were estimated at (24.58%, 5) and (33.91%, 3). In this case, the indexes estimated for PC_1 classified the measurement system as marginal. An analysis of PC_2 interpreted things differently, classifying the measurement system as unacceptable.

4.4. WPC

Steps 1 *and* 2: *assess and verify whether there are significant correlations between CTQs.*
Table 3 shows that all correlation coefficients are significant, so, to assess this measurement system, a multivariate GR&R study must be conducted (proceed to step 3.1).
Step 3.1: *create* **WPC** *vector*
As with the PCA method, an analysis was initially made of the principal components of the R_a, R_y, R_z, R_q, and R_t responses, using the correlation matrix of the data. The eigenvalues and eigenvectors obtained from the correlation matrix are the same as those presented in Table 5. **WPC** was obtained by weighting the principal component scores using Eq. (13).
Step 4: *estimate variance components*
WPC was adjusted for a two-way analysis of variance, according to the model in Eq. (14). The vector **WPC** represents the original set of responses $\mathbf{R_a}$, $\mathbf{R_y}$, $\mathbf{R_z}$, $\mathbf{R_q}$, and $\mathbf{R_t}$. The interaction term was not significant for a significance level of 0.05. Thus, the model can be adjusted to Eq. (20). Using Eqs. (17)–(19), (21) and (22), the study calculated variances for parts, repeatability, reproducibility (operators), measurement system, and total variance.
Step 5: *estimate multivariate indexes for assessing the measurement system*
Table 2 shows the square roots of these variances and, with Eqs. (23) and (24), the $\%R\&R_m$ and ndc_m indexes. $\%R\&R_m = 25.52\%$ classifies the measurement system as marginal. The index $ndc = 5$ states that the measurement system was able to identify five distinct categories of parts.

In summary, this numerical example has shown that ANOVA and PCA methods were unable to provide a single evaluation for the measurement process; the analyst was not capable of classifying the measurement system. WPC and MANOVA methods provided a single evaluation of the measurement system; their estimates, however, of the $\%R\&R_m$ index were divergent. Due to these findings, a simulation study was conducted to examine several scenarios involving measurement systems (unacceptable, marginal, and acceptable) and correlation structures (very low, low, medium, high, and very high) between CTQs.

Table 4
Eigenvalues of matrices $\hat{\boldsymbol{\Sigma}}_{\mathbf{P}}$, $\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}}$, and $\hat{\boldsymbol{\Sigma}}_{\mathbf{T}}$ are obtained by MANOVA.

	λ_1	λ_2	λ_3	λ_4	λ_5
Part	4.381	0.607	0.046	0.004	0.003
Measurement system	0.298	0.064	0.012	0.005	0.000
Total variation	4.655	0.685	0.066	0.012	0.003

Table 5
Principal component analysis for roughness parameters.

	Principal components				
	PC_1	PC_2	PC_3	PC_4	PC_5
Eigenvalue	4.312	0.638	0.037	0.011	0.002
Proportion	0.862	0.128	0.007	0.002	0.000
Cumulative	0.862	0.990	0.997	1.000	1.000
Responses			Eigenvectors		
R_z	0.475	0.052	−0.839	−0.008	0.259
R_y	0.446	0.457	0.282	−0.716	−0.018
R_t	0.439	0.501	0.263	0.697	−0.031
R_q	0.449	−0.448	−0.012	0.009	−0.773
R_a	0.425	−0.581	0.383	0.030	0.578

5. Simulation

5.1. Detailing the simulation study

The numerical example given above analyzes only a single case where the measurement system evaluated by each CTQ pertained to either a marginal or an unacceptable region and the correlation structure of variables was able to explain 86.2% to the first eigenvalue. The purpose of this simulation is to

Table 6
Mean vectors and variance-covariance matrices used to generate simulated data with different correlations and measurement systems (MS).

Scenarios	Mean vectors										Variance-covariance matrix
	P_1O_1	P_2O_1	P_3O_1	P_4O_1	P_5O_1	P_1O_2	P_2O_2	P_3O_2	P_4O_2	P_5O_2	
1	4.00	8.00	6.00	10.00	5.00	4.10	8.10	5.90	9.90	4.90	$\begin{bmatrix} 1.10 & 1.27 & 1.39 & 1.50 \\ 1.27 & 1.50 & 1.63 & 1.76 \\ 1.39 & 1.63 & 1.80 & 1.92 \\ 1.50 & 1.76 & 1.92 & 2.10 \end{bmatrix}$
Very Low corr.	8.00	6.00	13.00	9.00	11.00	7.90	6.10	12.90	9.10	10.90	
Unacceptable MS	9.00	10.00	13.00	16.00	7.00	9.10	10.10	12.90	15.90	7.10	
	7.00	11.00	5.00	10.00	15.00	7.10	10.90	5.10	10.10	15.10	
2	4.00	8.00	6.00	10.00	5.00	4.10	8.10	5.90	9.90	4.90	$\begin{bmatrix} 1.10 & 1.27 & 1.39 & 1.50 \\ 1.27 & 1.50 & 1.63 & 1.76 \\ 1.39 & 1.63 & 1.80 & 1.92 \\ 1.50 & 1.76 & 1.92 & 2.10 \end{bmatrix}$
Low corr.	8.00	7.00	9.00	12.00	11.00	7.90	6.90	9.10	12.10	10.90	
Unacceptable MS	9.00	10.00	7.00	13.00	15.00	9.10	10.10	6.90	13.10	14.90	
	7.00	13.00	11.00	14.00	17.00	7.10	13.10	11.10	13.90	16.90	
3	9.00	7.00	5.00	12.00	10.00	9.01	6.99	5.01	12.01	9.99	$\begin{bmatrix} 1.10 & 1.27 & 1.39 & 1.50 \\ 1.27 & 1.50 & 1.63 & 1.76 \\ 1.39 & 1.63 & 1.80 & 1.92 \\ 1.50 & 1.76 & 1.92 & 2.10 \end{bmatrix}$
Medium corr.	8.00	7.00	9.00	12.00	11.00	7.99	6.99	9.01	12.01	10.99	
Unacceptable MS	9.00	10.00	7.00	13.00	15.00	9.01	10.01	6.99	13.01	14.99	
	7.00	13.00	9.00	17.00	14.00	7.01	13.01	8.99	16.99	14.01	
4	6.00	4.00	8.00	10.00	12.00	6.01	4.01	7.99	9.99	12.01	$\begin{bmatrix} 1.50 & 1.58 & 1.63 & 1.67 \\ 1.58 & 1.70 & 1.73 & 1.78 \\ 1.63 & 1.73 & 1.80 & 1.83 \\ 1.67 & 1.78 & 1.83 & 1.90 \end{bmatrix}$
High corr.	3.00	6.00	9.00	11.00	15.00	3.01	6.01	9.01	10.99	14.99	
Unacceptable MS	6.00	8.00	11.00	15.00	13.00	6.01	8.01	11.10	15.10	13.10	
	8.00	10.00	12.00	16.00	14.00	7.99	10.01	12.01	16.01	14.01	
5	4.00	6.00	8.00	10.00	12.00	4.01	6.01	7.99	9.99	12.01	$\begin{bmatrix} 1.10 & 1.27 & 1.39 & 1.50 \\ 1.27 & 1.50 & 1.63 & 1.76 \\ 1.39 & 1.63 & 1.80 & 1.92 \\ 1.50 & 1.76 & 1.92 & 2.10 \end{bmatrix}$
Very high corr.	5.00	7.00	9.00	11.00	13.00	5.01	7.01	9.01	10.99	12.99	
Unacceptable MS	6.00	8.00	10.00	12.00	14.00	6.01	8.01	9.99	11.99	13.99	
	8.00	10.00	12.00	14.00	16.00	7.99	10.01	12.01	14.01	15.99	
6	4.00	8.00	6.00	10.00	5.00	4.10	8.10	5.90	9.90	4.90	$\begin{bmatrix} 0.22 & 0.25 & 0.28 & 0.30 \\ 0.25 & 0.30 & 0.33 & 0.35 \\ 0.28 & 0.33 & 0.36 & 0.38 \\ 0.30 & 0.35 & 0.38 & 0.42 \end{bmatrix}$
Very Low corr.	8.00	6.00	13.00	9.00	11.00	7.90	6.10	12.90	9.10	10.90	
Marginal MS	5.00	8.00	9.00	14.00	12.00	5.10	8.10	8.90	13.90	12.10	
	7.00	13.00	5.00	10.00	17.00	7.10	13.10	5.10	10.10	16.90	
7	6.00	8.00	4.00	11.00	10.00	6.10	8.10	3.90	10.90	9.90	$\begin{bmatrix} 0.22 & 0.25 & 0.28 & 0.30 \\ 0.25 & 0.30 & 0.33 & 0.35 \\ 0.28 & 0.33 & 0.36 & 0.38 \\ 0.30 & 0.35 & 0.38 & 0.42 \end{bmatrix}$
Low corr.	8.00	7.00	9.00	12.00	11.00	7.90	6.90	9.10	12.10	10.90	
Marginal MS	7.00	13.00	10.00	11.00	15.00	7.10	13.10	9.90	11.10	14.90	
	9.00	11.00	14.00	13.00	17.00	9.10	10.90	14.10	12.90	16.90	
8	9.00	7.00	5.00	12.00	10.00	9.01	6.99	5.01	12.01	9.99	$\begin{bmatrix} 0.22 & 0.25 & 0.28 & 0.30 \\ 0.25 & 0.30 & 0.33 & 0.35 \\ 0.28 & 0.33 & 0.36 & 0.38 \\ 0.30 & 0.35 & 0.38 & 0.42 \end{bmatrix}$
Medium corr	8.00	7.00	9.00	12.00	11.00	7.99	6.99	9.01	12.01	10.99	
Marginal MS	9.00	10.00	7.00	13.00	15.00	9.01	10.01	6.99	13.01	14.99	
	7.00	13.00	9.00	17.00	14.00	7.01	13.01	8.99	16.99	14.01	
9	6.00	4.00	8.00	10.00	12.00	6.01	4.01	7.99	9.99	12.01	$\begin{bmatrix} 0.22 & 0.25 & 0.28 & 0.30 \\ 0.25 & 0.30 & 0.33 & 0.35 \\ 0.28 & 0.33 & 0.36 & 0.38 \\ 0.30 & 0.35 & 0.38 & 0.42 \end{bmatrix}$
High corr	3.00	6.00	9.00	11.00	15.00	3.01	6.01	9.01	10.99	14.99	
Marginal MS	6.00	8.00	11.00	15.00	13.00	6.01	8.01	11.10	15.10	13.10	
	8.00	10.00	12.00	16.00	14.00	7.99	10.01	12.01	16.01	14.01	
10	4.00	6.00	8.00	10.00	12.00	4.01	6.01	7.99	9.99	12.01	$\begin{bmatrix} 0.22 & 0.25 & 0.28 & 0.30 \\ 0.25 & 0.30 & 0.33 & 0.35 \\ 0.28 & 0.33 & 0.36 & 0.38 \\ 0.30 & 0.35 & 0.38 & 0.42 \end{bmatrix}$
Very high cor	5.00	7.00	9.00	11.00	13.00	5.01	7.01	9.01	10.99	12.99	
Marginal MS	6.00	8.00	10.00	12.00	14.00	6.01	8.01	9.99	11.99	13.99	
	8.00	10.00	12.00	14.00	16.00	7.99	10.01	12.01	14.01	15.99	
11	4.00	8.00	6.00	10.00	5.00	4.10	8.10	5.90	9.90	4.90	$\begin{bmatrix} 0.04 & 0.04 & 0.05 & 0.05 \\ 0.04 & 0.05 & 0.05 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.07 \end{bmatrix}$
Very low corr	8.00	6.00	13.00	9.00	11.00	7.90	6.10	12.90	9.10	10.90	
Acceptable MS	5.00	8.00	9.00	14.00	12.00	5.10	8.10	8.90	13.90	12.10	
	7.00	13.00	5.00	10.00	17.00	7.10	13.10	5.10	10.10	16.90	
12	6.00	8.00	4.00	11.00	10.00	6.01	8.01	3.99	10.99	9.99	$\begin{bmatrix} 0.04 & 0.04 & 0.05 & 0.05 \\ 0.04 & 0.05 & 0.05 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.07 \end{bmatrix}$
Low corr	7.00	5.00	9.00	13.00	11.00	6.99	4.99	9.01	13.01	10.99	
Acceptable MS	7.00	13.00	10.00	11.00	15.00	7.01	13.01	9.99	11.01	14.99	
	6.00	10.00	14.00	12.00	17.00	6.01	9.99	14.01	12.01	16.99	
13	9.00	7.00	5.00	12.00	10.00	9.01	6.99	5.01	12.01	9.99	$\begin{bmatrix} 0.04 & 0.04 & 0.05 & 0.05 \\ 0.04 & 0.05 & 0.05 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.07 \end{bmatrix}$
Medium corr	8.00	7.00	9.00	12.00	11.00	7.99	6.99	9.01	12.01	10.99	
Acceptable MS	9.00	10.00	7.00	13.00	15.00	9.01	10.01	6.99	13.01	14.99	
	7.00	13.00	9.00	17.00	14.00	7.01	13.01	8.99	16.99	14.01	
14	6.00	4.00	8.00	10.00	12.00	6.01	4.01	7.99	9.99	12.01	$\begin{bmatrix} 0.04 & 0.04 & 0.05 & 0.05 \\ 0.04 & 0.05 & 0.05 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.07 \end{bmatrix}$
High corr.	3.00	6.00	9.00	11.00	15.00	3.01	6.01	9.01	10.99	14.99	
Acceptable MS	6.00	8.00	11.00	15.00	13.00	6.01	8.01	11.10	15.10	13.10	
	8.00	10.00	12.00	16.00	14.00	7.99	10.01	12.01	16.01	14.01	
15	4.00	6.00	8.00	10.00	12.00	4.01	6.01	7.99	9.99	12.01	$\begin{bmatrix} 0.04 & 0.04 & 0.05 & 0.05 \\ 0.04 & 0.05 & 0.05 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.06 \\ 0.05 & 0.06 & 0.06 & 0.07 \end{bmatrix}$
Very high corr.	5.00	7.00	9.00	11.00	13.00	5.01	7.01	9.01	10.99	12.99	
Acceptable MS	6.00	8.00	10.00	12.00	14.00	6.01	8.01	9.99	11.99	13.99	
	8.00	10.00	12.00	14.00	16.00	7.99	10.01	12.01	14.01	15.99	

evaluate other possible situations in a multivariate analysis of a measurement system and to compare the results achieved, primarily, through multivariate methods. Simulated data is generated for measurement systems that are unacceptable ($\%R\&R_m > 30\%$), marginal ($10\% < \%R\&R_m < 30\%$) and acceptable ($\%R\&R_m < 10\%$), as well as correlations that are very low ($\%PC_1 \le 65\%$), low ($65\% < \%PC_1 \le 75\%$), medium ($75\% < \%PC_1 \le 85\%$), high ($85\% < \%PC_1 \le 95\%$), and very high ($\%PC_1 > 95\%$). There are a total of 15 scenarios and 1800 simulated measurements. $\%PC_1$ is the result obtained from $\lambda_1 / \sum_{i=1}^{q} \lambda_i$. Simulated data were generated from the information in Table 6, according to the ssame amount of CTQs, parts, operators and replicates in Majeske (2008), $q=4$, $p=5$, $o=2$, and $r=3$. The data for the 15 simulated scenarios are shown in the appendix, in Tables A1–A5.

5.2. Criterion of methods' assessment

The numerical example showed that the ANOVA method might not be satisfactory when multiple CTQs are evaluated with significant correlations. Therefore, this simulation study will focus only on the comparison of multivariate methods. The $\%R\&R_m$ and $\%R\&R$ indexes were calculated to compare the methods. For each scenario, this study tried to obtain close $\%R\&R$ index values for CTQ_1, CTQ_2, CTQ_3, and CTQ_4. Thus, the indexes obtained by multivariate methods are expected to be close to those obtained by the ANOVA method. The criterion used in this work to determine if the estimated multivariate index, $\%R\&R_m$, is correct and is based on confidence intervals for means calculated from results obtained by the ANOVA method. The lower (LCL) and upper (UCL) limits of the confidence intervals are calculated by the following equations:

$$\mathrm{LCL} = \overline{\mathrm{CTQ}} - t_{N-1,\alpha/2} \frac{s}{\sqrt{N}} \tag{31}$$

$$\mathrm{UCL} = \overline{\mathrm{CTQ}} + t_{N-1,\alpha/2} \frac{s}{\sqrt{N}} \tag{32}$$

where $\overline{\mathrm{CTQ}}$ is the mean of $\%R\&R$ between CTQ_1, CTQ_2, CTQ_3 and CTQ_4; s is the standard deviation; N is the sample size and, and $t_{N-1,\alpha}$ is the $(1-\alpha)100$th percentile of a t distribution with $(N-1)$ degrees of freedom. Note that it would make little sense to evaluate situations where CTQs determine distinct classifications of the measurement system. For instance, CTQ_1 and CTQ_2 classify the measurement system as unacceptable and, on the other hand, CTQ_3 and CTQ_4 classify the measurement system as acceptable. In such situations, the confidence interval would be wider, and thus, $\%R\&R_m$ indexe would be easily estimated inside the limits.

5.3. Result analysis

Table 7 presents the results of calculations of the $\%R\&R$ index as well as the mean value and the 95% confidence interval, obtained by the ANOVA method. Table 8 shows the calculation results of the mean value, 95% confidence interval and $\%R\&R_m$ index, obtained by the PCA, MANOVA, and WPC methods. The analysis and comparison will be performed by the intra- and inter-methods. The intra-method analysis will provide an overview of the methods' performance to estimate the $\%R\&R_m$ index. The inter-method analysis will seek to justify the methods' deviations of estimates of the $\%R\&R_m$ index from the confidence intervals.

The intra-method analysis verified that, in the estimation of the $\%R\&R_m$ index, the WPC was more robust than MANOVA and PCA. The MANOVA method was able to estimate the multivariate index within the confidence interval only in scenarios S9, S11, and S14. Wang and Chien (2010) evaluated only the principal components with a cumulative percentage of explanation of at least 95% for the original variables. Thus, the PCA method was capable of estimating the multivariate index within the confidence interval only in scenarios S5, S10, S11, and S15. As seen in Table 8, the WPC method estimated $\% \ R\&R_m$ index within the confidence interval for all 15 scenarios.

For the inter-method analysis by PCA, Table 8 presents the analysis of measurement systems simulated for the four principal components. The values in parentheses show the explanation percentage of variability of the CTQs for each principal component. What should be evaluated are the scores of principal components that represent at least 95% of cumulative variability for CTQs. Therefore, scenarios with correlation structure

- Very low, low and medium: PC_1, PC_2, and PC_3 were analyzed.
- High: PC_1 and PC_2 were analyzed.
- Very high: only PC_1 was analyzed.

In S5, S10 and S15, only PC_1 was analyzed, the results showed that PCA was capable of estimating $\%R\&R_m$ within the confidence interval. For the other scenarios, PC_1 could not adequately explain the variability of the CTQs. Thus, when other principal

Table 7
Results for calculations of the $\%R\&R$ index, mean and 95% confidence interval.

Scenario			Univariate (%R&R)				Mean CI		
S	MS	Correlation	CTQ_1	CTQ_2	CTQ_3	CTQ_4	Mean	LCL	UCL
S1	Unacceptable	Very low	49.9	39.3	38.3	34.1	40.42	29.69	51.14
S2		Low	42.2	55.5	44.3	39.8	45.44	34.42	56.47
S3		Medium	40.8	52.4	42.6	36.9	43.18	32.63	53.72
S4		High	45.3	33.2	41.2	47.8	41.86	31.70	52.03
S5		Very high	31.1	34.9	37.8	41.1	36.21	29.45	42.97
S6	Marginal	Very low	15.8	14.1	13.7	10.2	13.48	9.75	17.21
S7		Low	18.6	27.2	21.3	24.1	22.82	16.95	28.69
S8		Medium	15.5	23.7	17.0	14.6	17.69	11.16	24.21
S9		High	13.2	10.3	13.6	16.9	13.50	9.19	17.80
S10		Very high	15.2	19.0	19.7	20.9	18.70	14.80	22.59
S11	Acceptable	Very low	8.4	6.3	4.9	5.3	6.22	3.67	8.77
S12		Low	5.6	4.6	6.7	5.4	5.54	4.15	6.92
S13		Medium	6.2	9.6	6.6	5.9	7.07	4.37	9.76
S14		High	5.7	4.5	5.9	7.3	5.84	4.00	7.69
S15		Very high	6.5	7.6	8.6	9.2	7.95	6.07	9.83

Table 8
Results for calculations of the mean, 95% confidence interval and %$R\&R_m$ index.

S	MEAN CI			MULTIVARIATE (%$R\&R_m$)					
	Mean	LCL	UCL	PC_1	PC_2	PC_3	PC_4	MANOVA	WPC
S1	40.42	29.69	51.14	52.24[a](55.8[b])	19.55 (29.1)	15.32 (14.1)	**42.08 (1.0)**	10.78	**39.71**
S2	45.44	34.42	56.47	**53.84 (70.7)**	10.48 (17.5)	20.91 (8.1)	19.32 (3.7)	13.30	**52.84**
S3	43.18	32.63	53.72	**47.79 (79.4)**	11.21 (9.4)	17.65 (7.5)	9.64 (3.7)	11.32	**47.87**
S4	41.86	31.70	52.03	**44.38 (88.3)**	8.94 (8.4)	29.62 (3.1)	75.92 (0.2)	28.15	**44.03**
S5	36.21	29.45	42.97	**36.10 (99.7)**	97.05 (0.2)	100.00 (0.0)	100.00 (0.0)	64.09	**36.11**
S6	13.48	9.75	17.21	18.46 (44.9)	6.92 (29.9)	2.79 (23.5)	32.43 (1.7)	4.97	**12.65**
S7	22.82	16.95	28.69	**24.97 (66.2)**	2.77 (17.8)	8.37 (15.9)	46.89 (0.2)	10.04	**26.98**
S8	17.69	11.16	24.21	**19.71 (79.8)**	6.01 (9.2)	**11.78 (7.5)**	7.19 (3.5)	5.40	**19.86**
S9	13.50	9.19	17.80	**14.17 (89.8)**	6.01 (7.6)	**9.99 (2.5)**	54.07 (0.1)	**14.31**	**14.00**
S10	18.70	14.80	22.59	**18.63 (99.9)**	93.87 (0.1)	100.00 (0.0)	100.00 (0.0)	47.23	**18.63**
S11	6.22	3.67	8.77	**6.41 (45.5)**	**6.39 (35.9)**	**4.59 (17.4)**	15.83 (1.1)	**4.08**	**4.10**
S12	5.54	4.15	6.92	**6.69 (67.6)**	1.15 (18.3)	2.25 (13.8)	**5.33 (0.4)**	2.01	**6.89**
S13	7.07	4.37	9.76	**7.87 (79.7)**	1.95 (8.9)	**5.75 (7.8)**	3.08 (3.6)	2.28	**8.04**
S14	5.84	4.00	7.69	**5.99 (90.3)**	3.34 (7.1)	3.96 (2.6)	33.24 (0.1)	**7.22**	**5.91**
S15	7.95	6.07	9.83	**7.92 (100.0)**	96.68 (0.0)	100.00 (0.0)	100.00 (0.0)	39.35	**7.92**

[a] %R&R.
[b] $\lambda_i / \sum_{j=1}^{q} \lambda_j \quad i = 1,2,\ldots,q$

Table 9
%$R\&R_m$ index for the inter-method analysis by MANOVA.

S	Mean	LCL	UCL	$\sqrt{\lambda MS_1/\lambda_{T1}}$	$\sqrt{\lambda MS_2/\lambda_{T2}}$	$\sqrt{\lambda MS_3/\lambda_{T3}}$	$\sqrt{\lambda MS_4/\lambda_{T4}}$	$\left(\prod_{i=1}^{4} \sqrt{\lambda MS_1/\lambda_{Ti}}\right)^{(1/4)}$
S1	40.42	29.69	51.14	**49.07[a](61.2[b])**	3.29 (27.4)	5.13 (10.5)	16.34 (0.9)	10.78
S2	45.44	34.42	56.47	**45.90 (74.6)**	9.35 (13.9)	8.32 (7.7)	8.76 (3.9)	13.30
S3	43.18	32.63	53.72	**45.97 (80.4)**	5.81 (10.8)	7.80 (5.2)	7.87 (3.6)	11.32
S4	41.86	31.70	52.03	**43.30 (87.3)**	52.70 (8.6)	14.66 (3.1)	18.78 (1.1)	28.15
S5	36.21	29.45	42.97	**37.75 (99.8)**	51.77 (0.2)	98.97 (0.0)	92.13 (0.0)	64.09
S6	13.48	9.75	17.21	**15.94 (53.4)**	2.14 (29.0)	2.40 (16.3)	7.43 (1.3)	4.97
S7	22.82	16.95	28.69	**27.39 (67.7)**	4.44 (18.3)	3.37 (13.9)	**24.73 (0.2)**	10.04
S8	17.69	11.16	24.21	**18.15 (81.2)**	3.00 (11.0)	4.45 (4.6)	3.51 (3.2)	5.40
S9	13.50	9.19	17.80	**14.50 (90.4)**	**14.24 (6.9)**	**10.03 (2.6)**	20.27 (0.1)	**14.31**
S10	18.70	14.80	22.59	**16.92 (99.9)**	56.75 (0.0)	70.15 (0.0)	73.86 (0.0)	47.23
S11	6.22	3.67	8.77	**7.27 (50.4)**	2.80 (34.5)	2.21 (34.5)	**6.14 (14.2)**	**4.08**
S12	5.54	4.15	6.92	**6.23 (71.5)**	0.95 (15.5)	0.73 (12.7)	3.80 (0.3)	2.01
S13	7.07	4.37	9.76	**7.11 (81.5)**	1.34 (10.6)	1.69 (4.6)	1.68 (3.3)	2.28
S14	5.84	4.00	7.69	**6.59 (90.8)**	**7.39 (6.4)**	**4.10 (2.8)**	13.63 (0.1)	**7.22**
S15	7.95	6.07	9.83	**7.78 (100.0)**	38.19 (0.0)	98.44 (0.0)	82.04 (0.0)	39.35

[a] %R&R.
[b] $\lambda_i / \sum_{j=1}^{q} \lambda_{Tj} \quad i = 1,2,\ldots,q.$

components were analyzed, the %$R\&R_m$ index was estimated outside the confidence interval (except for S11). In short, when the correlation structure between the CTQs requires that other principal components be analyzed, in addition to PC_1, the PCA method may fail (see Table 8).

For the inter-method analysis by MANOVA, Table 9 shows how the %$R\&R_m$ index was estimated for the 15 simulated scenarios. This verifies that this method is capable of estimating the multivariate index within of the confidence interval only in S9, S11, and S14. This index was obtained by MANOVA using the geometric mean of $\sqrt{\lambda_{MS}/\lambda_T}$ according to the amount of quality characteristics. This simulation study dealt with four characteristics. Thus, four eigenvalues of the $\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{T}}$ matrices were extracted. If the individual ratio $\sqrt{\lambda_{MS}/\lambda_T}$ for each pair of eigenvalues, 1, 2, 3, and 4, in $\hat{\boldsymbol{\Sigma}}_{\mathbf{MS}}$ and $\hat{\boldsymbol{\Sigma}}_{\mathbf{T}}$, provide different interpretations, the %$R\&R_m$ index estimated by MANOVA may not represent well the performance of the measurement system. Indeed, a geometric mean provides the same degree of importance in the analysis of each pair of eigenvalues. Nevertheless, it is known that the first eigenvalues wield a greater percentage of explaining the measured phenomenon than do the last eigenvalues. Therefore, it is confirmed that some form of weighting is needed for the calculation of this index.

In the inter-method analysis by WPC, Table 8 shows that the %$R\&R_m$ index was estimated within the confidence interval for the 15 simulated scenarios. WPC was more robust than PCA and MANOVA because it overcame some shortcomings of these methods. For PCA, when PC_1 is insufficient to explain all the variability of CTQs, other principal components can provide evaluations for the measurement system outside the confidence interval. MANOVA provides a single interpretation for the measurement system; however, the strategy of using geometric mean was not satisfactory. In WPC, the strategy of weighting the scores of principal components by their eigenvalues proved to be sufficient to correct the aforementioned shortcomings. Moreover, the simulation study showed that in using the WPC method the higher correlations between the CTQs, the closer to the mean value will be the estimates of $\%R\&R_m$.

6. Conclusions

This article has addressed the multivariate analysis of measurement systems through studies of such systems' repeatability and reproducibility. The article's main contribution is its proposal of a new method of multivariate analysis of measurement systems by weighting the principal components. To prove the efficiency of the method, this study evaluated measurements of the roughness parameters, obtained from AISI 12L14 steel turning machined with carbide tools. Additionally, simulated data were generated with different correlation structures for measurement systems considered acceptable, marginal, and unacceptable. The results obtained by the WPC method were compared to those obtained by the univariate and multivariate methods (PCA and MANOVA). Statistical analysis provided the following conclusions:

1. The numerical example showed that univariate analysis could not be satisfactory when correlated characteristics are measured with the same measuring instrument. Multivariate statistical techniques should be used so that a single classification represents the original set of variables.
2. As in the univariate method, through PCA, the individual analysis of the principal components may provide different interpretations. Therefore, PCA should be used only in cases where the correlations between the responses are very high. In this case, the first component represents reasonably well the variability of the measurement system.
1. The MANOVA method uses geometric mean to estimate the multivariate index to evaluate the measurement system. This approach may be incorrect when the ratio $\sqrt{\lambda_{MS}/\lambda_T}$ for each q pair of eigenvalues provides significant difference in their calculations.

   Taking the numerical example and the simulation study into account, WPC was the most robust method for assessing a multivariate measurement system. WPC was able to overcome such shortcomings as follows: providing a single assessment for all CTQs in a multivariate GR&R study; estimating the multivariate $\%R\&R_m$ index inside the confidence interval even when the correlation structure of CTQs is considered very low; and providing a strategy of weighting that guarantees greater importance for principal components most statistically significant to estimating the $\%R\&R_m$ index. Moreover, in scenarios where the CTQs showed high correlations, the estimates of the $\%R\&R_m$ indexes converged to the mean values calculated using the univariate method.

Acknowledgments

The authors would like to thank FAPEMIG, CAPES and CNPq for their support in this research. Additionally, the authors wish to gratefully acknowledge two referees of this paper, who helped clarify and improve its presentation.

Appendix A. Tables of simulated data

See Tables A1–A5.

Table A1
Very low correlation structure ($\%PC_1 \leq 65\%$).

			Unacceptable MS				Marginal MS				Acceptable MS			
i	*j*	*k*	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4
1	1	1	5.538	9.709	10.890	9.219	4.075	8.222	5.264	7.387	3.906	7.920	4.897	6.814
1	1	2	3.128	6.789	7.708	5.679	3.193	7.154	3.968	6.024	4.126	8.140	5.158	7.222
1	1	3	2.876	6.555	7.363	5.490	3.763	7.822	4.654	6.697	3.774	7.703	4.686	6.639
2	1	1	5.837	3.170	7.206	8.053	8.214	6.285	8.295	13.336	8.079	6.058	8.106	13.072
2	1	2	7.640	5.731	10.003	10.862	7.321	5.212	7.190	12.097	7.933	5.932	7.921	12.917
2	1	3	8.571	6.913	10.870	11.712	8.567	6.465	8.530	13.695	7.961	5.962	7.886	12.975
3	1	1	5.413	12.160	12.104	4.125	7.091	14.198	10.368	6.512	6.165	13.165	9.207	5.178
3	1	2	4.201	10.662	10.389	2.418	6.780	13.872	10.007	6.052	6.081	13.090	9.116	5.122
3	1	3	5.669	12.598	12.655	4.261	6.781	13.871	9.979	6.017	5.921	12.920	8.863	4.879
4	1	1	10.662	9.811	16.773	10.670	9.330	8.265	13.108	9.058	10.224	9.228	14.260	10.325
4	1	2	10.404	9.416	16.651	10.298	9.742	8.678	13.638	9.635	10.310	9.407	14.449	10.467
4	1	3	9.608	8.324	15.287	9.021	9.763	8.773	13.792	9.848	9.872	8.861	13.807	9.814
5	1	1	3.947	9.790	5.879	13.478	5.051	11.065	11.957	17.046	4.912	10.919	11.837	16.917
5	1	2	4.190	9.759	5.784	13.710	4.956	10.960	12.057	17.036	4.966	10.938	11.947	16.953
5	1	3	6.072	12.334	8.586	16.727	5.066	10.909	12.068	17.152	4.797	10.808	11.795	16.759
1	2	1	4.766	9.159	10.127	8.056	3.756	7.443	4.633	6.516	4.053	7.835	5.027	7.065
1	2	2	2.024	5.570	6.463	4.259	3.985	7.833	4.959	7.002	4.059	7.843	5.072	7.075
1	2	3	4.390	8.427	9.557	7.534	4.008	7.963	5.067	7.162	3.973	7.703	4.884	6.858
2	2	1	7.671	5.607	9.859	10.379	8.063	6.176	8.205	13.152	8.106	6.100	8.129	13.131
2	2	2	6.666	4.136	8.352	8.864	7.939	5.934	7.807	12.832	8.192	6.223	8.220	13.184
2	2	3	8.262	6.197	9.863	10.757	7.988	5.986	7.948	12.912	8.089	6.052	8.134	13.115
3	2	1	6.761	13.812	13.840	6.094	5.964	13.060	8.990	5.154	5.702	12.638	8.651	4.833
3	2	2	4.377	10.786	10.656	3.249	6.533	13.650	9.766	5.975	6.135	13.229	9.212	5.402
3	2	3	6.915	14.002	14.330	6.943	6.183	13.243	9.223	5.478	6.120	13.178	9.169	5.407
4	2	1	11.090	10.530	17.462	11.613	9.304	8.410	13.075	9.301	10.023	9.275	14.055	10.290
4	2	2	11.454	10.666	17.796	12.487	10.230	9.431	14.277	10.567	10.311	9.634	14.424	10.691
4	2	3	9.665	9.424	15.995	9.890	9.867	9.103	13.846	9.995	9.841	9.049	13.849	10.064
5	2	1	3.695	9.600	5.580	13.181	4.929	10.823	11.996	16.958	4.502	10.488	11.565	16.362
5	2	2	4.081	9.811	6.053	13.843	5.218	11.483	12.639	17.492	4.717	10.685	11.886	16.684
5	2	3	7.038	13.274	9.822	18.050	5.315	11.394	12.588	17.417	5.117	11.230	12.409	17.208

Table A2
Low correlation structure (65% < %PC_1 ≤ 75%).

			Unacceptable MS				Marginal MS				Acceptable MS			
i	*j*	*k*	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4
1	1	1	4.111	8.485	9.390	7.375	6.162	8.161	7.242	9.449	6.104	7.130	7.107	6.077
1	1	2	5.920	10.437	11.364	9.693	6.531	8.619	7.630	9.668	6.228	7.215	7.276	6.288
1	1	3	3.316	7.063	7.962	6.031	5.833	7.645	6.592	8.714	6.067	7.067	7.067	6.124
2	1	1	8.022	7.102	10.208	12.981	7.675	6.695	12.728	10.740	8.148	5.189	13.168	10.278
2	1	2	5.945	4.415	7.258	9.933	8.280	7.315	13.260	11.498	7.837	4.835	12.818	9.823
2	1	3	8.082	7.230	10.019	13.398	8.688	7.809	13.847	11.986	7.739	4.684	12.636	9.585
3	1	1	5.860	8.892	7.076	10.945	4.470	9.620	10.687	14.649	3.769	8.697	9.638	13.624
3	1	2	5.732	8.444	6.483	10.300	4.579	9.647	10.710	14.957	4.380	9.433	10.394	14.481
3	1	3	6.835	9.596	7.943	12.477	4.771	9.770	10.868	15.007	3.901	8.878	9.855	13.844
4	1	1	11.266	13.409	14.749	15.907	11.281	12.333	11.273	13.417	10.966	12.933	10.980	11.963
4	1	2	8.556	10.584	11.049	12.259	11.313	12.253	11.366	13.294	11.340	13.402	11.400	12.468
4	1	3	8.730	10.571	11.089	12.379	11.224	12.239	11.258	13.393	10.710	12.688	10.668	11.606
5	1	1	5.153	10.948	15.041	16.903	10.920	12.059	16.124	18.322	9.891	10.842	14.843	16.876
5	1	2	4.558	10.561	14.473	16.663	10.579	11.686	15.759	17.937	9.802	10.785	14.771	16.730
5	1	3	5.571	11.624	15.685	18.012	9.600	10.577	14.529	16.538	10.270	11.340	15.365	17.410
1	2	1	1.800	5.166	6.420	3.782	6.066	7.829	6.972	8.949	5.966	6.922	6.943	5.927
1	2	2	3.465	7.036	8.305	6.282	6.199	8.049	7.142	9.112	6.142	7.134	7.143	6.190
1	2	3	3.303	7.105	8.304	5.912	6.255	8.194	7.320	9.397	5.605	6.565	6.510	5.498
2	2	1	8.574	7.474	10.905	13.869	7.456	6.083	12.310	9.990	8.035	4.994	13.062	10.005
2	2	2	8.113	7.162	10.133	13.273	7.888	6.612	12.809	10.528	7.874	4.812	12.831	9.841
2	2	3	8.289	7.095	10.185	13.283	7.668	6.194	12.454	10.088	7.956	4.932	12.945	9.960
3	2	1	7.486	11.012	9.079	13.041	3.589	8.690	9.482	13.641	3.950	8.946	9.946	13.952
3	2	2	6.373	10.026	7.643	12.287	4.858	10.088	10.964	15.368	4.217	9.230	10.263	14.255
3	2	3	7.430	10.788	8.873	13.483	3.696	8.849	9.555	13.868	3.954	9.023	9.938	14.007
4	2	1	10.321	12.407	13.529	14.065	11.801	13.202	12.357	14.389	11.026	12.997	11.006	11.990
4	2	2	11.961	14.210	15.575	16.235	10.363	11.367	10.368	11.981	11.540	13.655	11.687	12.775
4	2	3	10.353	12.665	13.752	14.612	10.415	11.439	10.458	12.175	11.103	13.156	11.173	12.170
5	2	1	5.404	11.448	15.828	17.433	9.659	10.764	14.521	16.487	9.922	10.831	14.882	16.868
5	2	2	4.163	9.997	13.888	15.780	10.311	11.440	15.414	17.573	10.394	11.483	15.514	17.554
5	2	3	4.937	11.134	15.385	17.390	9.521	10.570	14.611	16.509	10.134	11.117	15.181	17.124

Table A3
Medium correlation structure (75% < %PC_1 ≤ 85%).

			Unacceptable MS				Marginal MS				Acceptable MS			
i	*j*	*k*	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4
1	1	1	7.362	6.254	7.015	4.843	8.554	7.567	8.542	6.526	9.137	8.187	9.171	7.176
1	1	2	9.505	8.687	9.497	7.605	9.157	8.262	9.253	7.239	8.850	7.855	8.846	6.754
1	1	3	8.763	7.739	8.566	6.810	8.926	7.831	8.729	6.716	9.139	8.174	9.168	7.117
2	1	1	7.026	7.037	10.072	13.008	6.397	6.376	9.214	12.230	6.877	6.800	9.843	12.775
2	1	2	7.405	7.609	10.454	13.739	6.967	7.043	9.867	12.949	7.282	7.353	10.415	13.410
2	1	3	7.130	7.030	9.959	13.183	7.063	7.174	10.140	13.143	6.858	6.815	9.794	12.758
3	1	1	6.522	10.962	8.853	11.050	4.675	8.687	6.530	8.540	5.049	9.097	7.079	9.097
3	1	2	4.887	8.604	6.848	8.522	5.091	9.068	7.136	9.178	4.748	8.708	6.668	8.620
3	1	3	5.334	9.349	7.335	9.238	4.842	8.750	6.843	8.818	4.925	8.886	6.918	8.898
4	1	1	10.844	10.396	11.165	15.282	11.896	11.936	12.965	16.928	12.027	12.027	13.092	17.041
4	1	2	12.521	12.695	13.827	17.694	11.526	11.436	12.571	16.388	12.096	12.142	13.130	17.182
4	1	3	12.133	12.381	13.226	17.017	12.795	12.912	13.843	18.042	12.335	12.359	13.445	17.505
5	1	1	8.756	9.568	13.183	12.027	10.210	11.207	15.233	14.192	9.933	10.943	14.927	13.953
5	1	2	9.734	10.728	14.719	13.962	10.412	11.463	15.412	14.623	10.174	11.244	15.214	14.268
5	1	3	9.176	9.969	13.858	12.961	9.778	10.729	14.600	13.599	10.119	11.104	15.138	14.166
1	2	1	10.509	9.705	10.849	8.890	8.920	7.917	8.892	6.928	8.988	7.958	9.020	6.972
1	2	2	9.214	8.467	9.357	7.684	9.124	8.105	9.125	6.983	8.904	7.807	8.860	6.833
1	2	3	8.921	7.893	8.658	6.894	9.822	8.938	9.920	8.014	8.588	7.463	8.420	6.371
2	2	1	6.388	6.283	9.432	12.575	6.974	6.881	9.990	12.912	7.187	7.202	10.255	13.329
2	2	2	6.671	6.771	9.691	12.697	7.563	7.648	10.779	13.802	7.171	7.194	10.252	13.205
2	2	3	6.077	6.154	8.919	11.895	6.503	6.361	9.296	12.272	6.776	6.790	9.838	12.745
3	2	1	5.946	10.287	8.191	10.299	4.304	8.157	6.027	7.898	4.990	8.962	6.950	8.919
3	2	2	6.812	11.074	9.315	11.516	5.615	9.752	7.858	9.867	5.123	9.164	7.160	9.180
3	2	3	5.555	9.353	7.423	9.237	5.117	9.036	7.091	8.936	4.977	8.950	6.955	8.913
4	2	1	11.747	11.835	12.692	16.751	12.580	12.703	13.722	17.752	12.243	12.322	13.345	17.331
4	2	2	13.518	13.463	14.947	18.887	11.828	11.812	12.756	16.701	12.181	12.218	13.248	17.331
4	2	3	11.366	11.255	12.273	15.824	12.340	12.393	13.443	17.307	12.263	12.315	13.311	17.347
5	2	1	12.633	14.025	18.132	17.549	10.236	11.261	15.252	14.218	9.789	10.771	14.773	13.721
5	2	2	8.829	9.892	13.505	12.502	9.338	10.086	14.012	13.116	10.185	11.191	15.168	14.239
5	2	3	11.883	13.279	17.684	16.751	10.037	10.975	14.967	14.080	10.098	11.105	15.160	14.144

Table A4
High correlation structure (85% < %PC_1 ≤ 95%).

			Unacceptable MS				Marginal MS				Acceptable MS			
i	*j*	*k*	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4
1	1	1	5.927	2.974	5.798	7.966	5.861	2.819	5.865	7.720	6.345	3.358	6.428	8.414
1	1	2	7.848	5.044	8.006	10.053	5.634	2.604	5.637	7.534	6.204	3.261	6.314	8.307
1	1	3	6.771	3.665	6.807	8.825	5.809	2.792	5.712	7.652	5.839	2.838	5.825	7.851
2	1	1	4.957	7.096	9.025	11.111	3.878	5.801	7.917	9.838	4.054	6.020	7.997	10.013
2	1	2	4.105	5.695	7.928	9.614	4.607	6.680	8.705	10.680	3.720	5.638	7.609	9.624
2	1	3	5.962	8.030	10.125	12.332	3.674	5.766	7.675	9.700	3.976	5.974	7.970	10.001
3	1	1	9.663	10.694	12.706	13.857	8.294	9.252	11.332	12.378	7.886	8.869	10.807	11.795
3	1	2	9.125	10.272	12.202	13.080	8.116	9.276	11.343	12.220	8.153	9.157	11.216	12.211
3	1	3	8.496	9.783	11.970	12.824	7.505	8.518	10.275	11.287	8.062	9.095	11.094	12.117
4	1	1	9.613	10.900	14.776	15.729	9.225	10.052	14.033	14.903	9.926	10.868	14.863	15.893
4	1	2	10.055	11.140	15.028	16.147	10.765	11.886	15.978	16.942	10.191	11.229	15.259	16.285
4	1	3	10.461	11.672	15.652	16.588	10.495	11.629	15.557	16.622	9.858	10.781	14.760	15.774
5	1	1	12.044	14.981	12.968	13.936	12.245	15.285	13.293	14.364	12.085	15.068	13.104	14.096
5	1	2	10.434	13.544	11.527	12.411	11.652	14.689	12.525	13.471	11.842	14.845	12.830	13.807
5	1	3	12.389	15.447	13.295	14.265	11.580	14.634	12.625	13.557	11.981	14.966	12.998	13.981
1	2	1	5.627	2.902	6.034	7.877	6.056	3.066	6.004	8.114	6.122	3.161	6.137	8.124
1	2	2	5.182	2.053	5.297	6.857	5.470	2.528	5.418	7.410	6.283	3.322	6.264	8.337
1	2	3	6.199	3.260	6.182	8.411	6.085	3.132	6.076	8.067	6.056	3.096	6.086	8.108
2	2	1	3.820	6.037	7.657	9.884	4.757	7.033	9.073	11.178	4.128	6.163	8.159	10.200
2	2	2	3.117	4.893	7.158	8.976	3.593	5.572	7.556	9.536	3.878	5.819	7.833	9.781
2	2	3	5.243	7.408	9.637	11.622	4.422	6.401	8.647	10.675	4.296	6.370	8.410	10.404
3	2	1	6.483	7.117	9.155	9.991	7.644	8.622	10.695	11.740	7.928	8.952	10.974	11.950
3	2	2	6.881	7.992	9.898	10.893	7.850	8.937	11.005	11.791	8.109	9.147	11.266	12.245
3	2	3	7.418	8.725	11.047	11.689	7.743	8.764	10.719	11.724	8.009	9.076	11.114	12.041
4	2	1	11.784	12.658	17.130	18.033	9.925	10.929	15.012	15.928	9.815	10.815	14.953	15.875
4	2	2	8.024	8.551	12.753	13.888	9.663	10.704	14.785	15.721	9.555	10.460	14.563	15.418
4	2	3	7.990	8.713	12.642	13.857	9.865	10.906	14.969	15.930	9.755	10.704	14.794	15.699
5	2	1	11.706	14.730	12.825	13.754	11.689	14.676	12.658	13.697	12.024	14.999	13.131	14.068
5	2	2	9.981	12.905	10.666	11.810	11.430	14.276	12.363	13.217	12.048	14.983	13.096	14.019
5	2	3	9.712	12.616	10.555	11.213	11.469	14.330	12.264	13.061	11.960	14.973	13.058	13.936

Table A5
Very high correlation structure (%PC_1 > 95%).

			Unacceptable MS				Marginal MS				Acceptable MS			
i	*j*	*k*	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4	CTQ_1	CTQ_2	CTQ_3	CTQ_4
1	1	1	1.770	2.310	3.491	4.832	4.651	5.800	6.886	8.741	3.692	4.633	5.526	7.498
1	1	2	4.541	5.674	6.394	8.693	5.051	6.202	7.288	9.404	4.036	5.046	6.104	8.073
1	1	3	4.368	5.189	6.334	8.184	4.023	4.945	5.975	7.873	3.976	5.037	5.987	7.959
2	1	1	4.293	4.592	5.735	7.380	5.391	6.475	7.330	9.141	6.201	7.229	8.299	10.305
2	1	2	5.830	7.004	7.842	10.045	6.517	7.742	8.710	10.692	5.895	6.820	7.877	9.859
2	1	3	5.921	6.715	7.917	9.768	6.181	7.117	8.123	10.190	6.034	7.050	8.079	10.030
3	1	1	6.591	7.298	8.079	9.932	8.605	9.723	10.854	12.943	8.129	9.160	10.155	12.167
3	1	2	7.219	7.873	8.761	10.841	7.994	9.086	10.004	11.935	8.169	9.201	10.211	12.218
3	1	3	7.269	8.099	9.137	10.827	8.161	9.184	10.184	12.300	8.193	9.156	10.178	12.231
4	1	1	11.607	12.845	14.001	16.535	9.743	10.845	11.741	13.708	10.100	11.114	12.126	14.127
4	1	2	10.218	11.504	12.546	14.532	9.758	10.741	11.779	13.737	10.550	11.682	12.746	14.788
4	1	3	8.177	8.899	9.423	11.454	10.044	11.161	12.168	14.049	10.049	11.026	12.050	14.045
5	1	1	11.790	12.775	13.848	15.883	11.694	12.574	13.515	15.507	11.699	12.633	13.620	15.643
5	1	2	12.991	14.080	15.079	16.945	11.566	12.324	13.354	15.412	12.092	13.109	14.107	16.144
5	1	3	11.405	12.458	13.431	15.565	11.426	12.255	13.321	15.209	11.712	12.666	13.661	15.626
1	2	1	3.416	4.207	5.195	7.287	3.672	4.490	5.503	7.402	3.844	4.792	5.823	7.768
1	2	2	5.788	7.046	8.342	10.471	3.810	4.667	5.631	7.551	4.332	5.399	6.459	8.440
1	2	3	3.276	4.164	5.175	6.967	4.336	5.282	6.379	8.434	3.824	4.803	5.775	7.774
2	2	1	5.500	6.351	7.120	9.275	5.898	6.883	7.961	9.857	5.974	6.960	7.971	9.977
2	2	2	4.564	5.348	6.219	8.084	5.051	5.943	6.912	8.830	6.327	7.372	8.450	10.465
2	2	3	6.251	7.200	8.234	10.507	6.181	7.187	8.370	10.348	5.961	6.940	7.987	9.934
3	2	1	7.708	8.677	9.513	11.200	8.414	9.686	10.664	12.636	7.998	9.015	9.988	12.013
3	2	2	7.505	8.190	9.258	11.315	7.749	8.755	9.783	11.767	7.647	8.631	9.624	11.524
3	2	3	8.696	9.779	10.999	12.983	8.453	9.561	10.515	12.683	7.988	9.017	10.063	12.036
4	2	1	9.368	10.540	11.409	13.288	10.386	11.533	12.574	14.752	9.899	10.870	11.817	13.808
4	2	2	11.325	12.348	13.839	15.881	10.176	11.341	12.445	14.404	10.226	11.287	12.284	14.387
4	2	3	9.259	10.265	11.101	12.799	9.478	10.418	11.467	13.311	10.181	11.220	12.218	14.316
5	2	1	12.650	13.423	14.460	16.587	12.293	13.267	14.216	16.235	11.921	12.912	13.879	15.941
5	2	2	10.976	11.749	12.705	14.936	12.241	13.393	14.378	16.330	11.691	12.693	13.604	15.573
5	2	3	11.092	11.994	12.852	14.619	11.947	12.794	13.967	15.847	12.079	13.062	14.141	16.099

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